A story of geometry and fluctuations in the stage of condensates

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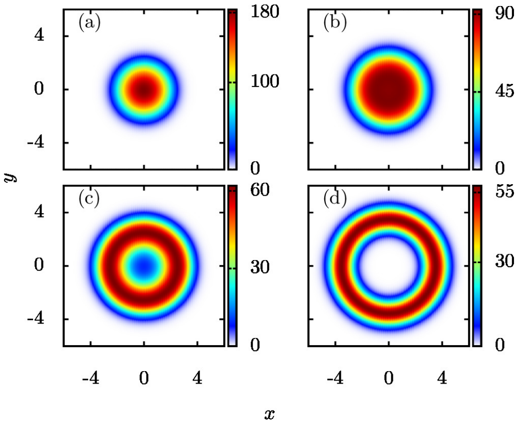
AR, D. Angom, arXiv:1511.08655 (2015)

Plan of the talk

Trapping potential

from simply to multiply connected



From pancake to toroidal

How does the fluctuations get modified?

Toroidal condensates

- Spontaneous seeding of topological defects through Kibble-Zurek mechanism
(Weiler *et al.* *Nature* **455**, 948 (2008))
- Observation of persistent superfluid flow
(Ryu *et al.*, *Phys. Rev. Lett.* **99**, 260401 (2007))

What is the basic nature of the fluctuations in toroidal condensates ?

can provide fundamental understanding on the scale, structure, and energetics of the defect formation.

W. H. Zurek, *Nature* **317**, 505 (1985)
T. W. B. Kibble, *J. Phys. A* **9**, 1387 (1976)

Gross-Pitaevskii equation (GPE)

- Equation of motion of the condensate wavefunction is given by Gross-Pitaevskii equation(GPE), strictly valid at $T = 0$K.

$$i\hbar\frac{\partial\psi}{\partial t} = \left[-\frac{\hbar^2}{2m}\nabla^2 + V_{\rm trap}(\mathbf{r}) + gN|\psi|^2\right]\psi,$$

  - $\psi \equiv \psi(\mathbf{r}, t)$: condensate wave function
  - $g = \dfrac{4\pi\hbar^2 a}{m}$
  - a: atomic scattering length > 0 : repulsive
  - N: Number of atoms in the condensate

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E. P. Gross, *Il Nuovo Cimento Series 10* **20**, (1961),
L. P. Pitaevskii, *Soviet Physics JETP-USSR* **13**, (1961),
C. Pethick & H. Smith, *Bose-Einstein Condensation in Dilute Gases*, (2008)

Many-body Hamiltonian

$$\hat{H} = \int \underbrace{d\mathbf{r}\, \hat{\psi}^\dagger(\mathbf{r},t) \left[\hat{h}(\mathbf{r}) - \mu\right] \hat{\psi}(\mathbf{r},t)}_{\text{single-particle part}}$$

$$+\frac{1}{2} \iint \underbrace{d\mathbf{r} d\mathbf{r}'\, \hat{\psi}^\dagger(\mathbf{r},t)\hat{\psi}^\dagger(\mathbf{r}',t) U(\mathbf{r}-\mathbf{r}')\hat{\psi}(\mathbf{r}',t)\hat{\psi}(\mathbf{r},t)}_{\text{two-particle interaction term}}$$

where $\hat{h} = K.E + V_{\mathrm{trap}}$

$$U(\mathbf{r}-\mathbf{r}') = g\delta(\mathbf{r}-\mathbf{r}'), \left\langle \int d\mathbf{r}\, \hat{\psi}^\dagger(\mathbf{r},t)\hat{\psi}(\mathbf{r},t) \right\rangle = N$$

U:: Repulsive contact interaction; N:: Total number of atoms

$$\left[\hat{\psi}(\mathbf{r}), \hat{\psi}(\mathbf{r}')\right] = \left[\hat{\psi}^\dagger(\mathbf{r}), \hat{\psi}^\dagger(\mathbf{r}')\right] = 0; \left[\hat{\psi}(\mathbf{r}), \hat{\psi}^\dagger(\mathbf{r}')\right] = \delta(\mathbf{r}-\mathbf{r}')$$

A.Griffin, *Phys. Rev. B* **53**, 9341 (1996)

Generalized GP equation

Equation of motion of the Bose field operator

$$i\hbar\frac{\partial\hat{\psi}(\mathbf{r},t)}{\partial t} = (\hat{h}-\mu)\hat{\psi}(\mathbf{r},t) + g\hat{\psi}^{\dagger}(\mathbf{r},t)\hat{\psi}(\mathbf{r},t)\hat{\psi}(\mathbf{r},t)$$

where, $\hat{\psi}(\mathbf{r},t) = \phi(\mathbf{r}) + \tilde{\psi}(\mathbf{r},t)$. $\phi/\tilde{\psi}$ is the condensate/fluctuation part.

Including the fluctuation terms, the generalized GP equation is

$$(\hat{h}-\mu)\phi(\mathbf{r}) + g|\phi(\mathbf{r})|^2\phi(\mathbf{r}) + \underbrace{2g\tilde{n}(\mathbf{r})\phi(\mathbf{r}) + g\tilde{m}(\mathbf{r})\phi^*(\mathbf{r})}_{T-dependent} = 0$$

using the *HFB approximation*.

- $\hat{h} = K.E. + V_{\rm trap}, \int |\phi(\mathbf{r})|^2\, d\mathbf{r} = 1$

A.Griffin, *Phys. Rev. B* **53**, 9341 (1996);
Hutchinson *et. al Phys. Rev. Lett.* **78**, 1842 (1997)

Bogoliubov de-Gennes equations

Equation of motion of the fluctuation operator

$$\begin{aligned} i\hbar\frac{\partial\tilde{\psi}}{\partial t} &= i\hbar\frac{\partial}{\partial t}(\hat{\psi}-\phi), \\ &= (\hat{h}-\mu)\tilde{\psi} + 2gn(\mathbf{r})\tilde{\psi} + gm(\mathbf{r})\tilde{\psi}^{\dagger}, \end{aligned}$$

where, $n(\mathbf{r}) = |\phi(\mathbf{r})|^2 + \tilde{n}(\mathbf{r})$; $m(\mathbf{r}) = \phi^2(\mathbf{r}) + \tilde{m}(\mathbf{r})$;
$\tilde{\psi} = \sum_j \left[u_j\hat{\alpha}_j e^{-iE_jt} - v_j^*\hat{\alpha}_j^{\dagger}e^{iE_jt}\right]$;
$u_j, v_j \Rightarrow$ quasiparticle amplitudes
Bogoliubov de-Gennes equations:

$$\begin{aligned} \mathscr{L}u_j &- gmv_j = E_ju_j \\ \mathscr{L}v_j &- gm^*u_j = -E_jv_j \end{aligned}$$

where $\mathscr{L} = \hat{h} - \mu + 2gn(\mathbf{r})$

Non-condensate density

Density of the thermal component:

$$\langle\tilde{\psi}^{\dagger}(\mathbf{r})\tilde{\psi}(\mathbf{r})\rangle = \tilde{n} = \sum_j \left\{ \left[|u_j|^2 + |v_j|^2\right] \langle\hat{\alpha}_j^{\dagger}\hat{\alpha}_j\rangle + |v_j|^2 \right\}.$$

and multiplying factor

$$\langle\hat{\alpha}_j^{\dagger}\hat{\alpha}_j\rangle = \frac{1}{e^{\beta E_j} - 1} \equiv N_0(E_j).$$

is the Bose-Einstein distribution.

At $T = 0$**,** $\tilde{n} = \sum_j |v_j|^2 \rightarrow$ **Quantum depletion**

The anomalous average

$$\langle\tilde{\psi}(\mathbf{r})\tilde{\psi}(\mathbf{r})\rangle = \tilde{m} = -\sum_j u_j v_j^* \left[2\langle\hat{\alpha}_j^{\dagger}\hat{\alpha}_j\rangle + 1\right],$$

is neglected in the *HFB-Popov approximation*.

A. Roy and D. Angom, *Phys. Rev. A* **90**, 023612 (2014)
A.Griffin, *Phys. Rev. B* **53**, 9341 (1996)

Quasiness

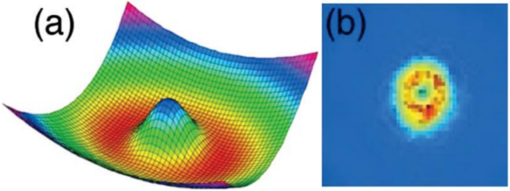
from harmonic to toroidal

$V(x,y,z) = (1/2)m\omega_x^2(x^2 + \alpha^2 y^2 + \lambda^2 z^2)$

Quasi-2D condition

$\omega_x, \omega_y \ll \omega_z$, and $\hbar\omega_z \gg \mu, k_\mathrm{B}T$.

$\alpha = \omega_y/\omega_x$ and $\lambda = \omega_z/\omega_x$, $U = 2a\sqrt{2\pi\lambda}$



Present Scheme:

$V_\mathrm{net}(x,y) = V_\mathrm{trap}(x,y) + U_0 e^{-(x^2+y^2)/2\sigma^2}$

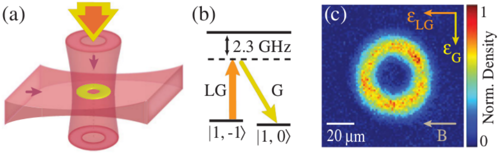
$U_0 = 0 \to$ Harmonic; $U_0 \gg 0 \to$ Toroid.

Ryu *et al.*, *Phys. Rev. Lett.* **99**, 260401 (2007)

Other Scheme:

Using Laguerre-Gaussian (LG_p^l) beams.

$p \geqslant 0$ and l, are radial and azimuthal orders of the laser beam.



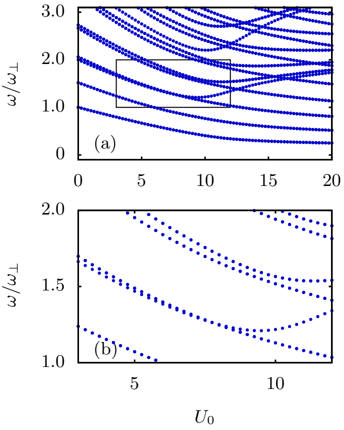
$$U_l(r) = U_1\sqrt{l}\left(\frac{r}{r_T}\right)^{2l} e^{-l(r^2/r_T^2-1)}$$

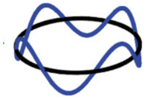
- LG_p^l laser beams *do not have* limiting case equivalent to a harmonic oscillator potential.
- LG_p^l laser beams *are not suitable* to examine the variation in fluctuations as a pancake shaped condensate is transformed to a toroidal one.

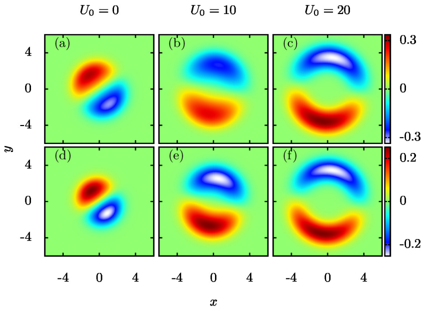
Ramanathan *et al.*, *Phys. Rev. Lett.* **106**, 130401 (2011)
Wright *et al.*, *Phys. Rev. A* **63**, 013608 (2000)

The Kohn or dipole mode

role of U_0







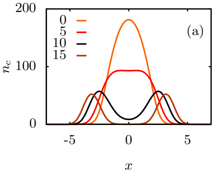
Moulder *et al.*, Phys. Rev. A **86**, 013629 (2012);
AR, D. Angom , arXiv:1511.08655 (2015) [under review].

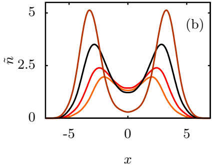
The Kohn or dipole mode
noticeable features

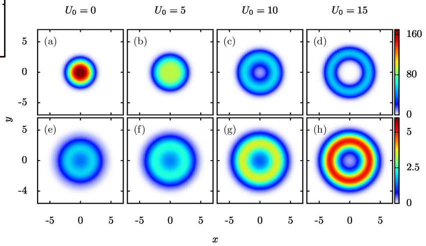
- When $U_0 = 0$, the quasiparticle spectrum has a Goldstone mode, and doubly degenerate Kohn modes with $\omega/\omega_\perp = 1$.
- For $U_0 \neq 0$, the condensate density shows a dip in the central region, and an overall increase in the radial extent due to the repulsive Gaussian potential.
- *The wavelength of excitations becomes longer as they now lie along the circumference of the toroid. This decreases the quasiparticle energies.*
- *Variation in energy of breathing ($l = 0$) and hexapole ($l = 3$) modes with increasing U_0.*

Quantum and thermal depletion

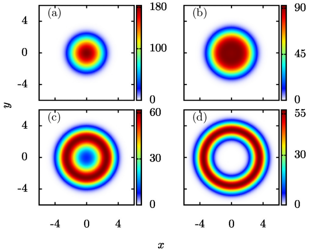
- *Enhancement of non-condensate density* $\tilde{n}$ due to quantum (thermal) fluctuations with increasing U_0 at $T = 0 (T \neq 0)$.
- At $T \neq 0$, the *condensate and thermal densities have coincident maxima* when $U_0 \gg 0$. This is in stark contrast to the case of pancake geometry ($U_0 = 0$).







Conclusions



How does the fluctuations get modified?

- We have demonstrated the *decrease in the energy of the Kohn mode* the external trapping potential undergoes transformation from a *simply to multiply connected geometry.*
- Close to the pancake to toroidal condensate transition, energies of all the modes with $l = 0$ increase.
- For a toroidal trap, at $T \neq 0$ *the condensate and the thermal densities have overlapping maxima.*

Thank You