Statics and dynamics of Bose-Einstein condensate

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Plan of the talk

Introduction

Gross-Pitaevskii equation

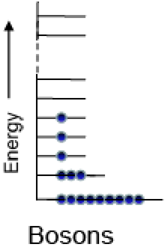
Effects of finite temperature on condensates

Binary Bose-Einstein condensate

Rayleigh-Taylor Instability(RTI)

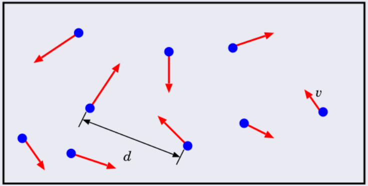
Bose-Einstein Condensation

- Macroscopic occupation of non-interacting bosons in the ground state of the system
- A gas of bosonic particles cooled below a critical temperature $T_c \approx nK$ condenses into an ideal Bose-Einstein condensate (BEC)
- De Broglie wavelength λ_{dB} comparable to the distance between the particles–wave packets start to overlap

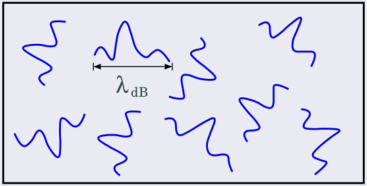


Basic Phenomenon

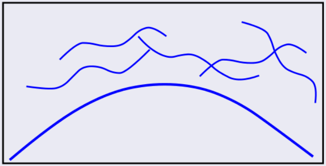
High Temperature (T), Density d^{-3}



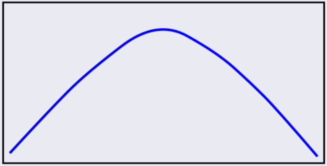
Low Temperature (T), λ_{dB}



Bose-Einstein Condensate, $T = T_{crit}$



Pure Bose Condensate, $T = 0$



P. Muruganandam (Workshop on HPC, PRL Ahmedabad, 2012).

Gross-Pitaevskii equation

Also referred to as **Non-linear Schrödinger equation**

- Equation of motion of the condensate wavefunction is given by Gross-Pitaevskii equation (GPE), strictly valid at $T = 0$K.

$$i\hbar \frac{\partial \psi}{\partial t} = \left[-\frac{\hbar^2}{2m}\nabla^2 + V_{\mathrm{trap}}(\mathbf{r}) + gN|\psi|^2 \right]\psi,$$

  - $\psi \equiv \psi(\mathbf{r}, t)$: condensate wave function
  - $g = \frac{4\pi\hbar^2 a}{m}$
  - a: *s-wave* scattering length > 0 : repulsive
  - N: Number of atoms in the condensate

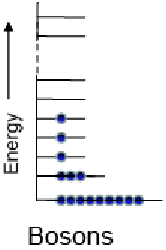
$$V_{\mathrm{trap}} = \frac{m}{2}\left(\omega_x^2 x^2 + \omega_y^2 y^2 + \omega_z^2 z^2\right)$$

E. P. Gross, *Il Nuovo Cimento Series 10*, **20**, (1961);
L. P. Pitaevskii, *Soviet Physics JETP-USSR*, **13**, (1961);
C. Pethick & H. Smith, *Bose-Einstein Condensation in Dilute Gases*, (2008)

Why do we study finite temperature effects?

Region of interest :: $0 < T < T_c$

- $T = 0K$ is physically unattainable. Experiments take place at finite temperatures.
- When $T \neq 0$, the condensate co-exists with the *thermal cloud*. Interactions between condensate and non-condensate (thermal) atoms cannot be neglected.



Generalized GPE

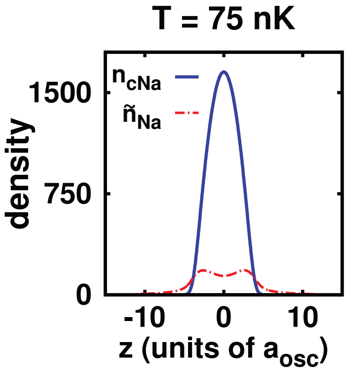
Including the thermal/non-condensate component term, the generalized GP equation is

$$(\hat{h} - \mu)\phi(\mathbf{r}) + g|\phi(\mathbf{r})|^2\phi(\mathbf{r}) + \underbrace{2g\tilde{n}(\mathbf{r})\phi(\mathbf{r})}_{T-dependent} = 0$$

where $\tilde{n}$ is the non-condensate density. Calculated from Bogoliubov-de-Gennes equations.

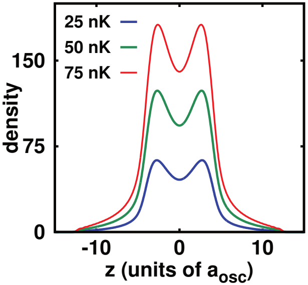
Bose-Einstein distribution function $\rightarrow \dfrac{1}{e^{\beta E_j} - 1}$.

- $\hat{h} = K.E. + V_{\mathrm{trap}}$



The noncondensate (dashed) and the condensate (solid) densities at $T = 75\,\mathrm{nK}$

Noncondensate density for 10 000 Sodium atoms at various temperatures



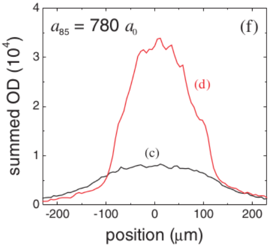
Unique feature of binary BEC

Role of interactions

Phase Separation

$U_{11} U_{22} - (U_{12})^2 > 0$

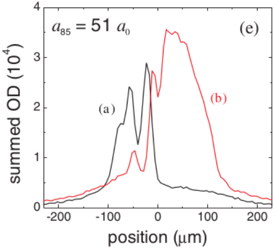
Miscible regime



^{85}Rb–^{87}Rb

$U_{11} U_{22} - (U_{12})^2 < 0$

Immiscible regime



Papp et. al, *Phys. Rev. Lett.*,**101**, (2008)

Coupled Generalized GP equation

$$\hat{h}_1\phi_1 \quad + \quad U_{11}\left[n_{1c} + 2\tilde{n}_1\right]\phi_1 + U_{12}[n_{2c} + \tilde{n}_2]\phi_1 = 0,$$
$$\hat{h}_2\phi_2 \quad + \quad U_{22}\left[n_{2c} + 2\tilde{n}_2\right]\phi_2 + U_{12}[n_{1c} + \tilde{n}_1]\phi_2 = 0.$$

- $n_{kc} = |\phi_k|^2$: Condensate density of $k^{\rm th}$ species
- $\tilde{n}_k$: Non-condensate density of $k^{\rm th}$ species

Dynamical evolution

Instabilities

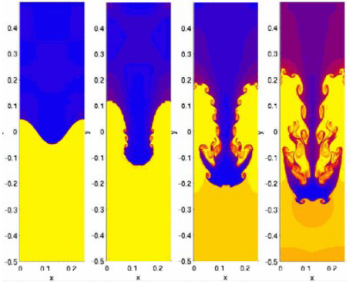
Instabilities in phase separated regime

- Rayleigh-Taylor instability
- Kelvin-Helmholtz instability

Sasaki et al. , *Phys. Rev. A* **80**, (2009);
S. Gautam and D. Angom , *Phys. Rev. A* **81**, (2010);
Takeuchi et. al , *Phys. Rev. B* **81**, (2010);
Kadokura et al. , *Phys. Rev. A* **85**, (2012);
AR, S. Gautam, D. Angom, arXiv:1210.0381, (2012)

Rayleigh-Taylor Instability(RTI)

- Instability of an interface when a lighter fluid supports a heavier one in a gravitational field
- Can also occur when a lighter fluid pushes a heavier one
- Leads to turbulent mixing of the two fluids as the perturbations at the interface grow exponentially



Courtesy:
en.wikipedia.org

P. G. Drazin & W. H. Reid, *Hydrodynamic Stability* (2004)

Thank You